

Highly magnetized super-Chandrasekhar white dwarfs and their consequences

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Stars with a stable magnetic field: from pre-main sequence to
compact remnants

August 28 to September 1, 2017, Brno, Czech Republic

The talk is based on the following papers

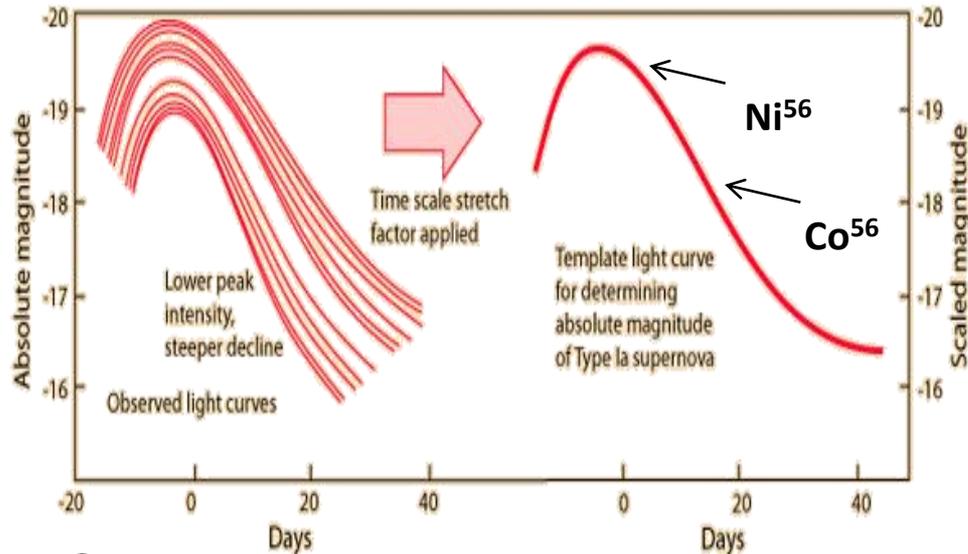
- MUKHOPADHYAY, A. R. Rao, T. S. Bhatia, MNRAS (press), 2017
- MUKHOPADHYAY, A. R. Rao, JCAP, 05, 007, 2016
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Flow-Chart of Evolution of our Idea

- ❑ Since last 5 years or so, **we have initiated** exploring highly magnetized super-Chandrasekhar white dwarfs (B-WDs), explaining peculiar type Ia supernovae: Over-luminous → Many groups joined us including a group from Prague
- ❑ Brings super-Chandrasekhar white dwarfs **in lime-light**
- ❑ Approach: (1) Spherical symmetric Newtonian model with **constant/fluctuating magnetic fields** → deformation effects speculated (2) Spherical symmetric general relativistic model with **realistic varying magnetic field** (3) Model capturing **self-consistent departure from spherical symmetry by general relativistic magnetohydrodynamic (GRMHD) analyses**: Result was already speculated in second paper

MOTIVATION OF INTRODUCING THIS NEW FIELD

TYPE Ia SUPERNOVAE



Champagne Supernova - SN 2003fg

Courtesy:

Georgia State University

PECULIARITIES

Not touched upon here

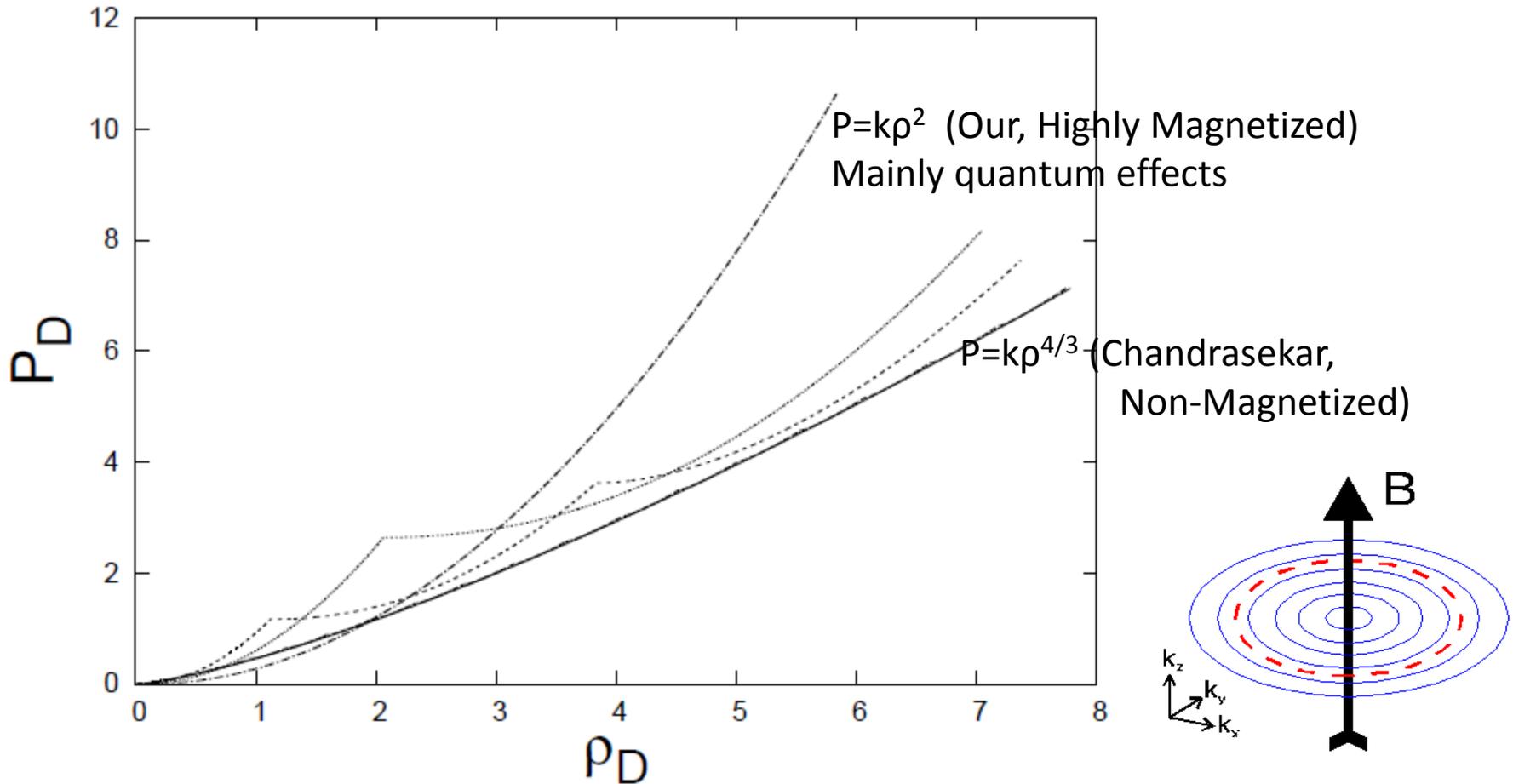
Over-luminous, very high Ni mass $\gtrsim 1.3M_{\odot}$, violates luminosity-stretch relation, very low ejecta velocity

SN 2006gz, SN 2007if, SN 2009dc, SN 2003fg - seem to suggest super-Chandrasekhar-mass white dwarfs ($2.1 - 2.8 M_{\odot}$) as their most likely progenitors (Hicken et al. 2007, Howell et al. 2006, Scalzo et al. 2010).

Under-luminous, very low Ni mass $\sim 0.1M_{\odot}$

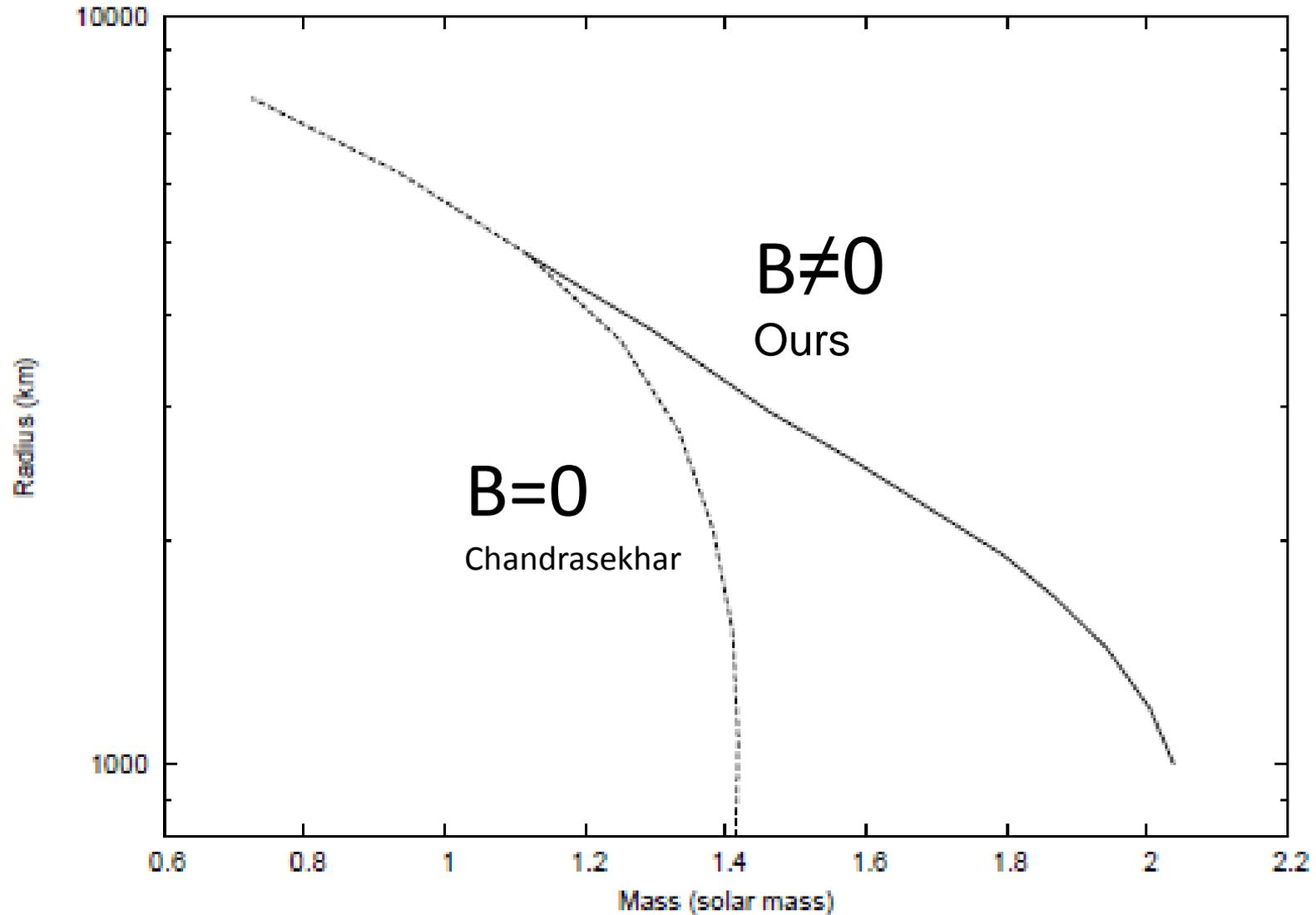
SN 1991bg, SN 1997cn, SN 1998de, SN 1999by, SN 2005bl (Filippenko et al. 1992, Mazzali et al. 1997, Taubenberger et al. 2008) – suggest sub-Chandrasekhar explosion

Simplest exploration: Constant/fluctuating magnetic field throughout for spherical white dwarfs → ideal case helping to understand in the spirit of Chandrasekhar's work → fluctuating length scale is similar to corresponding Compton wavelength → quantum mechanical effects



density in units of 2×10^9 gm/cc
 pressure in units of 2.668×10^{27} erg/cc

Mass-Radius Relation



Obtaining new limit: spirit of Chandrasekhar

$$\frac{1}{\rho} \frac{d}{dr} \left(P + \frac{B^2}{8\pi} \right) = F_g + \frac{\vec{B} \cdot \nabla \vec{B}}{4\pi\rho}, \quad \frac{dM}{dr} = 4\pi r^2 \rho$$

$$M \propto K^{3/2} \rho_c^{\frac{3-n}{2n}}, \quad R \propto K^{1/2} \rho_c^{\frac{1-n}{2n}} \quad P = K \rho^\Gamma$$

For extremely
high density regime

$$K = K_m \propto B_D^{-1} \propto \rho_c^{-2/3}$$

Mass is independent of ρ_c and radius becomes zero

Ours

Chandrasekhar's

$\Gamma=2$ and hence $n=1$

$\Gamma=4/3$ and hence $n=3$

$$M = \left(\frac{hc}{2G} \right)^{3/2} \frac{1}{(\mu_e m_H)^2} \approx \frac{10.312}{\mu_e^2} M_\odot,$$

$$M_{Ch} = \frac{\sqrt{6}}{32\pi} \left(\frac{hc}{G} \right)^{3/2} \left(\frac{2}{\mu_e} \right)^2 \frac{\xi_1^2 |\theta'(\xi_1)|}{m_H^2}$$

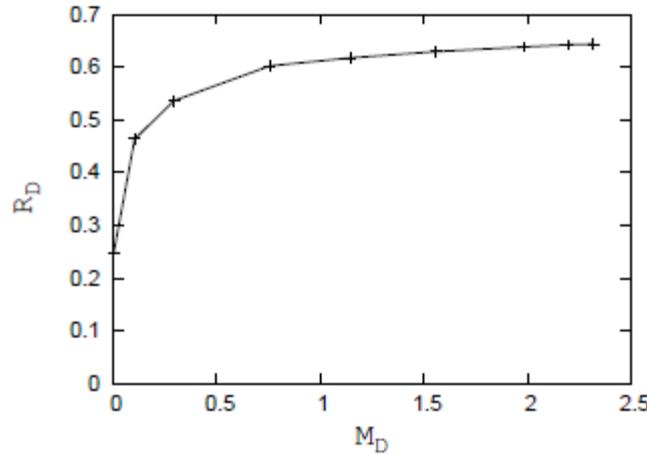
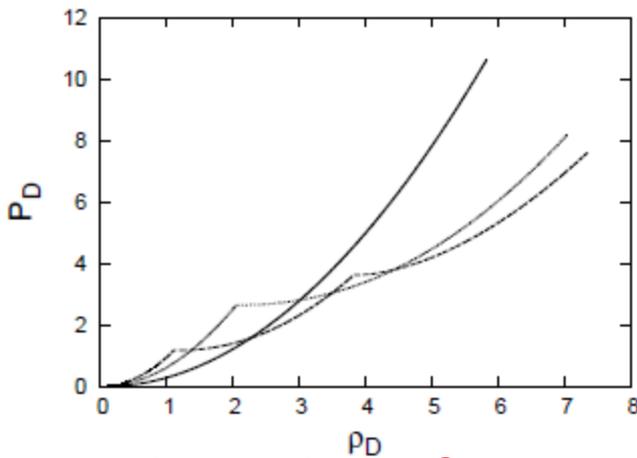
For $\mu_e=2$ (carbon-oxygen white dwarf)

$$M \approx 2.58 M_\odot.$$

$$1.44 M_\odot$$

Ideal versus Non-Ideal cases

- Limiting mass $M=2.58M_{\odot}$ with $B, \rho \rightarrow$ very high,
 $R \rightarrow$ very small
- Of course ideal result in the spirit of Chandrasekhar-limit
- For $B < 5 \times 10^{15} \text{G}$ and $\rho \sim 10^{10} \text{g/cc}$, $M=2.44M_{\odot}$, $R \sim 650 \text{km}$
- For $B=0$ (Chandrasekhar), $\rho \sim 10^{10} \text{g/cc}$, $M=1.39M_{\odot}$, $R \sim 1000 \text{km}$
- Worries of inverse β -decay, pycnonuclear fusion and general relativity related instabilities do not stand



Das, BM, PRD 2012
Das, BM, PRL 2013
Das, BM, MPLA 2014

P in units of $2.667 \times 10^{27} \text{ erg/cc}$, ρ in units of $2 \times 10^9 \text{ g/cc}$
 M in units of M_{\odot} , R in units of 1000 km

Self-consistent solutions with **varying magnetic field** without spherical approximation in **general relativity**

- The anisotropic effect due to a **strong magnetic field** may cause the shape of the white dwarfs to **deviate from spherical symmetry**. The problem would then consist of at least **two independent variables** instead of the single radial coordinate.
- In order to self-consistently take into account this departure from spherical symmetry, we have constructed equilibrium models of strongly magnetized, static, white dwarfs using the publicly available **General Relativistic Magnetohydrodynamic (GRMHD)** numerical codes **XNS** (Bucciantini & Del Zanna A&A 2011, Pili et al. MNRAS 2014) www.arcetri.astro.it/science/ahead/XNS/ and also **LORENE** (Bonazzola et al. 1993; Bocquet et al. 1995) <http://www.lorene.obspm.fr>.
- **XNS and LORENE** are well tested codes, before us used to compute axisymmetric equilibrium configurations of strongly magnetized neutron stars. We have applied the codes for **strongly magnetized white dwarfs**, with appropriate modifications, for the **first time in the literature**.

| $\Omega_c(\text{rad/s})$ | $M(M_\odot)$ | $r_c(\text{km})$ | $\Omega_{eq}(\text{rad/s})$ | $B_{max}(10^{14}\text{G})$ | KE/GE | ME/GE | r_p/r_c |
|--------------------------|--------------|------------------|-----------------------------|----------------------------|----------------------|-------|-----------|
| 0.003 | 1.878 | 1869 | 0.0002 | 3.0921 | 1.5×10^{-9} | 0.134 | 1.038 |
| 8.112 | 1.918 | 1922 | 0.458 | 3.1016 | 0.011 | 0.135 | 0.982 |
| 18.252 | 2.097 | 2118 | 0.843 | 3.1407 | 0.054 | 0.137 | 0.816 |
| 28.392 | 2.478 | 2454 | 0.988 | 3.2098 | 0.121 | 0.143 | 0.617 |

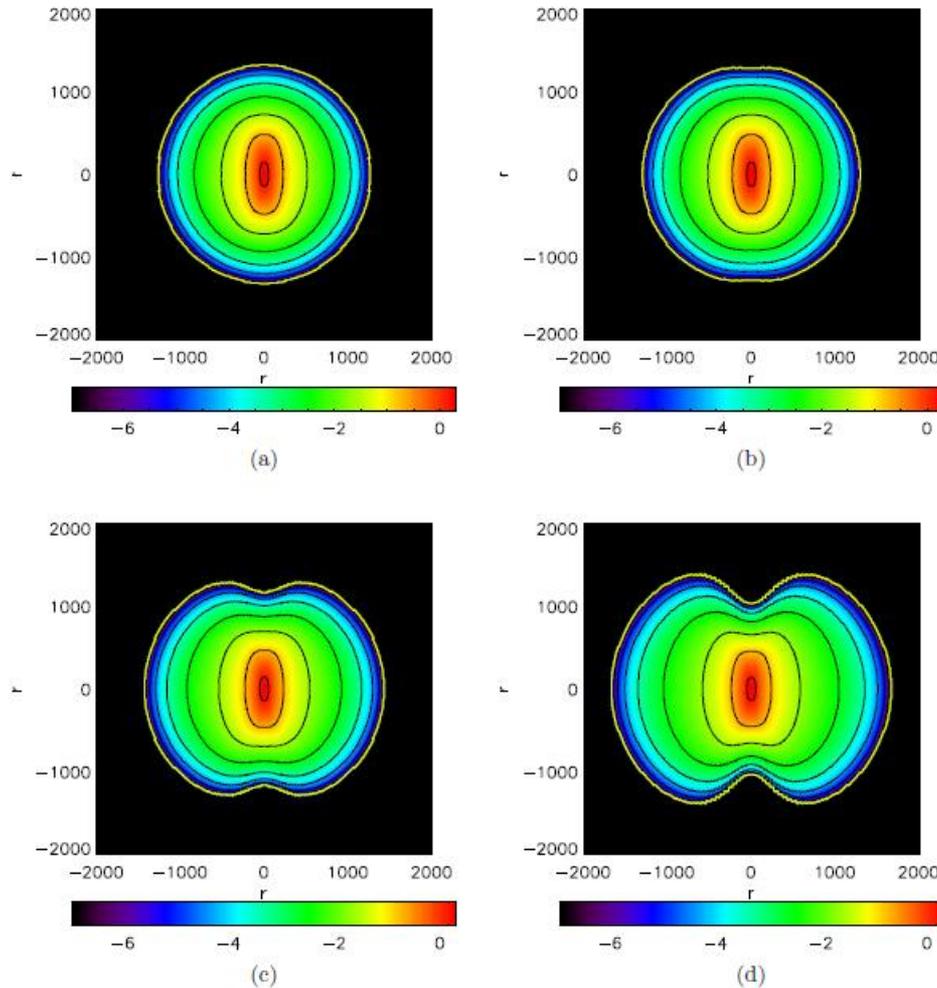


Figure 5.11: Sequence of differentially rotating configurations with a purely toroidal magnetic field, with changing Ω_c . The magnetic profile is fixed with $m = 1.4$ for the power law (4.4) and $B_{max} \approx 3.1 \times 10^{14}\text{G}$. The panels are contour plots of $\log\left(\frac{\rho}{\rho_0}\right)$ corresponding to the Ω_c values (a) 0.003 rad/s (b) 8.112 rad/s (c) 18.252 rad/s (d) 28.392 rad/s. The corresponding physical quantities are listed in the table above. The radial co-ordinate r is in units of 1.48km.

Rotating Magnetized White Dwarfs

ME/GE, KE/GE are in accordance with Ostriker & Hartwick 1968; Braithwaite 2009

Density contours for purely **toroidal** field configuration with varying central angular velocity and $A^2 \sim 10^5$

$$A^2(\Omega_c - \Omega) = \frac{(\Omega - \omega)r^2 \sin^2 \theta e^{2(\beta - \nu)}}{1 - (\Omega - \omega)^2 r^2 \sin^2 \theta e^{2(\beta - \nu)}}$$

$$P = k\rho^\Gamma, \quad \Gamma \approx 4/3, \quad v_m \geq 20$$

$$B_{max} \sim 3 \times 10^{14} \text{ G}, \quad B_s \geq 10^9 \text{ G}$$

Polar hollow

$$M \geq 2.5 M_\odot$$

$$\rho_0 = 10^{10} \text{ gm/cc}$$

Subramanian, BM, MNRAS 2015

| Ω_c | M | r_e | Ω_{eq} | B_{max} | KE/GE | ME/GE | r_p/r_e |
|------------|-------|-------|---------------|-----------|--------------------|-------|-----------|
| 2.028 | 1.502 | 1072 | 0.322 | 3.419 | 6×10^{-4} | 0.057 | 0.818 |
| 12.168 | 1.568 | 1072 | 1.929 | 3.574 | 0.020 | 0.06 | 0.769 |
| 24.336 | 1.798 | 1072 | 3.854 | 4.119 | 0.074 | 0.071 | 0.636 |
| 32.448 | 2.054 | 1037 | 5.429 | 4.755 | 0.117 | 0.080 | 0.556 |

Rotating Magnetized White Dwarfs

ME/GE, KE/GE are in accordance with Ostriker & Hartwick 1968; Braithwaite 2009

Density contours for purely **poloidal** field configuration with varying central angular velocity

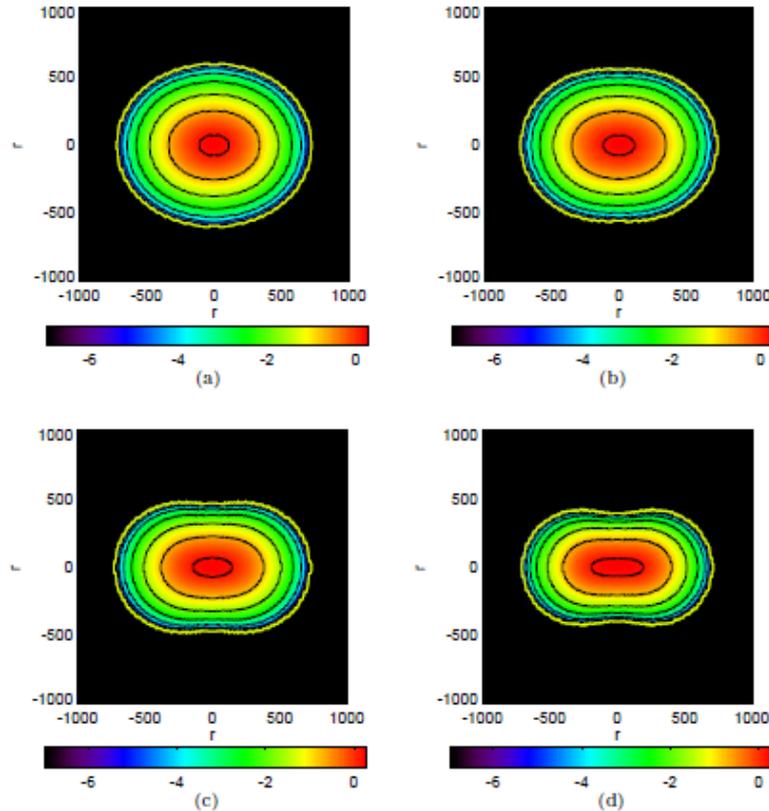


Figure 19. Sequence of differentially rotating configurations with a purely poloidal magnetic field with changing Ω_c and $B_{max} = 3.1$ fixed. The panels are contour plots of $\log\left(\frac{\rho}{\rho_0}\right)$ corresponding to the Ω_c values (a) 2.028, (b) 12.168, (c) 24.336, (d) 32.448. The corresponding physical quantities are listed in Table 9.

$$P = k\rho^\Gamma, \quad \Gamma \approx 4/3, \quad v_m \geq 20$$

$$B_{max} \sim 3 \times 10^{14} \text{ G}, \quad B_s \geq 10^9 \text{ G}$$

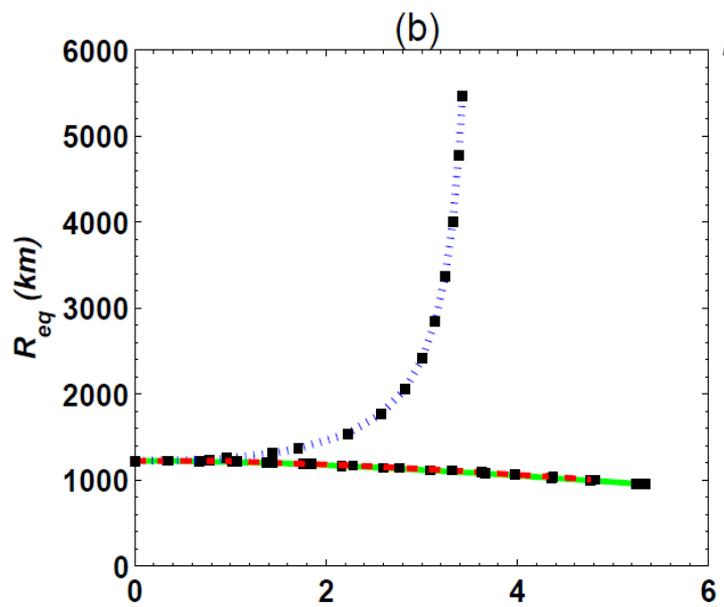
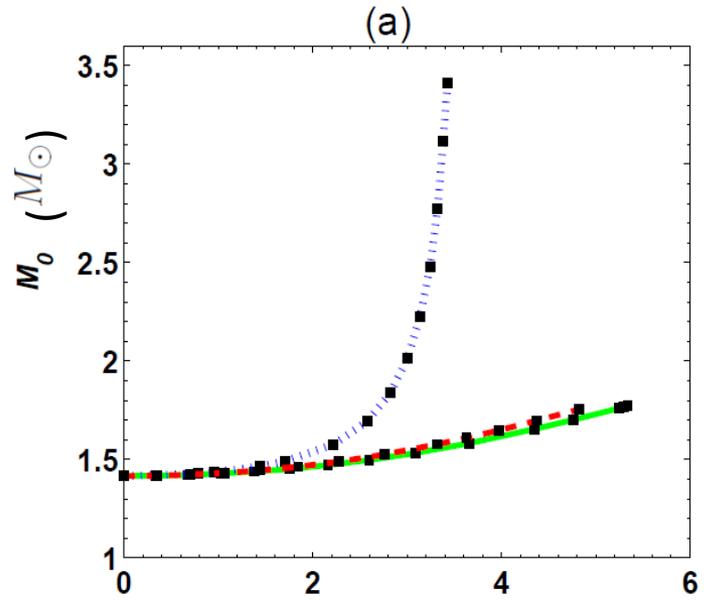
$$M \geq 2 M_\odot$$

$$\rho_0 = 10^{10} \text{ gm/cc}$$

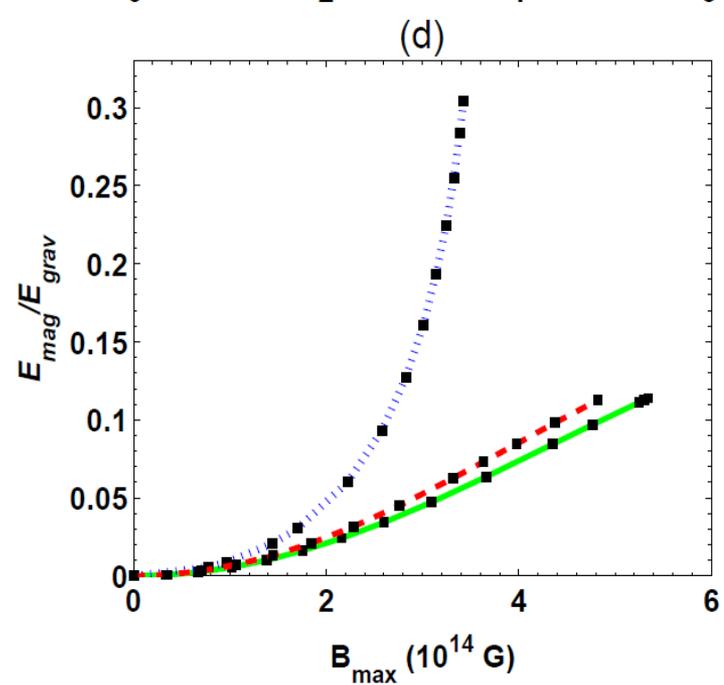
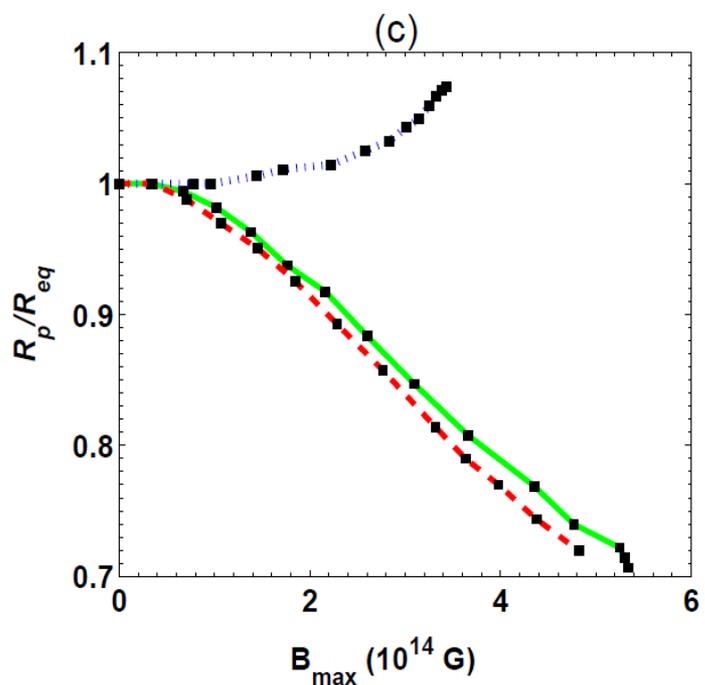
Subramanian, BM, MNRAS 2015

EQUILIBRIUM SEQUENCES OF MAGNETIZED WHITE DWARFS WITHOUT ROTATION WITH FIXED CENTRAL DENSITY

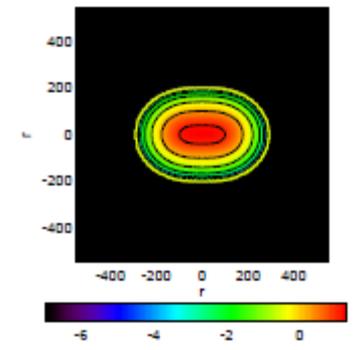
$$\rho_c = 2 \times 10^{10} \text{ gm/cm}^3$$



— Purely Poloidal
⋯ Purely Toroidal
- - - Mixed/Twisted Torus



Individual
■ White
 Dwarfs



Other aspects/implications

- Can explain SGRs/AXPs without invoking very strong fields, as required for neutron star based model
- Explaining white dwarf pulsars, e.g GCRT J1745-3009, AR Scorpii: as the seed of B-WD
- Candidates for gravitational wave (GW) search

Modeling SGR/AXP by B-WDs

Shortcoming of existing models

- Although very popular without proper alternatives, there are several shortcomings in magnetar model
- No observational evidence is for strongly magnetized neutron stars, as strong as required for magnetar model
- Fermi observations are inconsistent with high energy gamma-ray emissions in magnetars
- Inferred upper limit of B_p , e.g. for SGR 0418+5729, is quite smaller than field required to explain observed X-ray luminosity
- Hence, high magnetic dipole moment is not mandatory
- Weakly magnetized white dwarf based model (C-WDs: Paczynski, Usov, Rueda, Malheiro, Ruffini) is challenged by observed short spin periods and low UV-luminosities

Modeling compact stars as dipole

Assuming white dwarfs behaving as rotating magnetic dipoles: originally proposed by Paczynski, Usov in 1990s; later used by Malheiro, Rueda, Ruffini

Rate of energy loss

$$\dot{E}_{\text{rot}} = -\frac{\mu_0 \Omega^4 \sin^4 \alpha}{5\pi c^3} |m|^2,$$

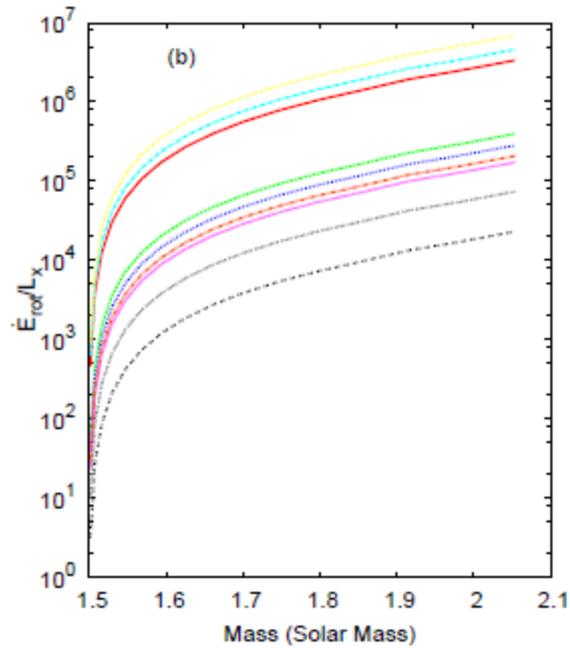
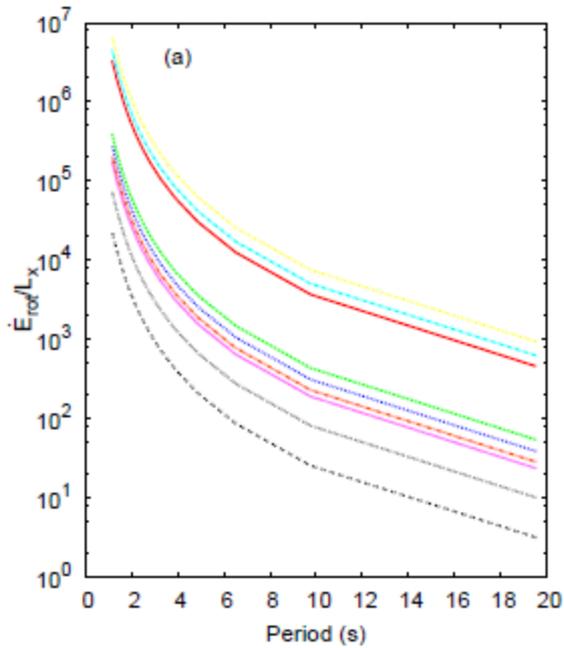
Dipole nature of magnetic field

$$B = \frac{\mu_0 |m|}{2\pi R^3},$$

$$I\Omega\dot{\Omega} = \dot{E}_{\text{rot}},$$

$$B_s = \sqrt{\frac{5c^3 I P \dot{P}}{4\pi^2 R^6 \sin^2 \alpha}} G$$

Neutron star based model cannot explain SGR/AXP as rotationally powered pulsar unless $B_s \sim 10^{15}$ G

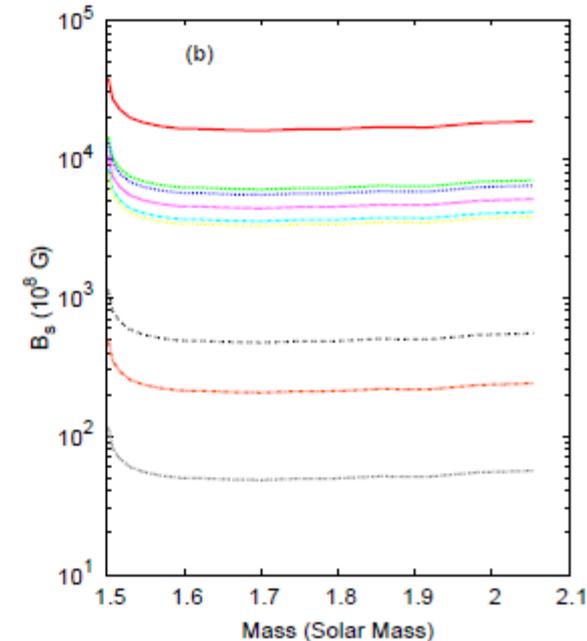
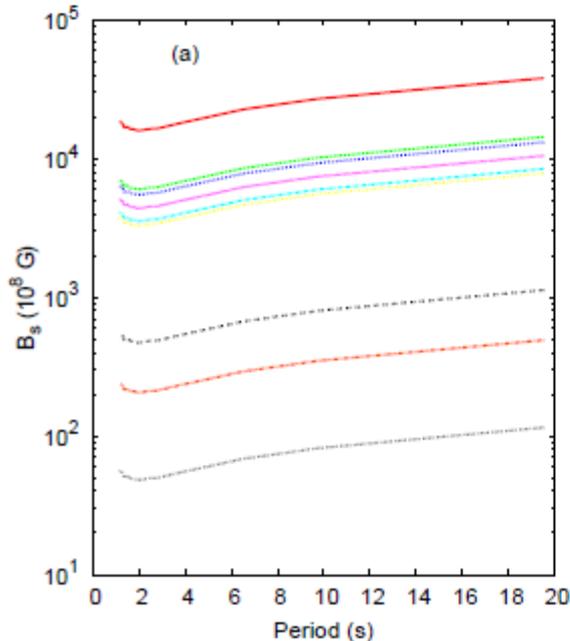


$$\dot{E}_{rot} = -\frac{\mu_0 \Omega^4 \sin^4 \alpha}{5\pi c^3} |m|^2,$$

$$100 \lesssim \dot{E}_{rot}/L_x \lesssim 10^7$$

correspond to 1E 1547-54, 1E 1048-59, SGR 1806-20, SGR 1900+14, SGR 0526-66, SGR 1822-1606, 1E 1841-045, SGR 0418+5729 and 1E 2259+586. For other details, see Table 1.

Rotational energy change is several orders of magnitude higher than observed X-ray luminosity



Explaining SGRs/AXPs as

$$100 \lesssim \dot{E}_{\text{rot}}/L_x \lesssim 10^7$$

BM, Rao, JCAP 2016

Rotationally Powered B-WDs → No problem with UV luminosity cutoff

| AXPs/SGRs | P (s) | \dot{P} (10^{-11}) | L_x (10^{35} ergs s^{-1}) | α (degree) | $L_{UV \text{ min}}$ (ergs s^{-1}) B-WD | $L_{UV \text{ min}}$ (ergs s^{-1}) C-WD |
|---------------|------------|-----------------------------|--------------------------------------|----------------------|--|--|
| 1E 1547-54 | 2.07 | 2.32 | 0.031 | 5 – 15 | 5.7×10^{28} | 4.8×10^{29} |
| 1E 1048-59 | 6.45 | 2.7 | 0.054 | 5 – 15 | 3.5×10^{26} | 9.2×10^{29} |
| 1E 1841-045 | 11.78 | 4.15 | 2.2 | 15 | 1.6×10^{28} | 1.7×10^{30} |
| 1E 2259+586 | 6.98 | 0.048 | 0.19 | 2 – 3 | 3.4×10^{26} | 1.5×10^{29} |
| SGR 1806-20 | 7.56 | 54.9 | 1.5 | 15 | 3.4×10^{26} | 3.5×10^{30} |
| SGR 1900+14 | 5.17 | 7.78 | 1.8 | 15 | 8.6×10^{28} | 1.3×10^{30} |
| SGR 0526-66 | 8.05 | 6.5 | 2.1 | 15 | 6.4×10^{27} | 1.7×10^{30} |
| SGR 0418+5729 | 9.08 | 5×10^{-4} | 6.2×10^{-4} | 1 – 5 | 3×10^{28} | 1.8×10^{29} |
| SGR 1822-1606 | 8.44 | 9.1×10^{-3} | 4×10^{-3} | 1 – 5 | 3.4×10^{26} | 8×10^{28} |

- More sources to be observed by AstroSat's wide band spectroscopic capabilities with cyclotron resonance energy $E = 11.6 (B/10^{12}\text{G})$ keV in the spectrum → confirming surface field strength
- Also other features: wide band spectral shape (like tail/second peak) can be examined in the context of beamed emission from the pole of a white dwarf, X-ray luminosity of SGRs/AXPs $\sim 10^{36}$ erg/sec

Continuous Gravity Wave Signal from B-WDs

Signal emitted by a tri-axial compact star rotating around a principle axis of inertia is characterized by the amplitude

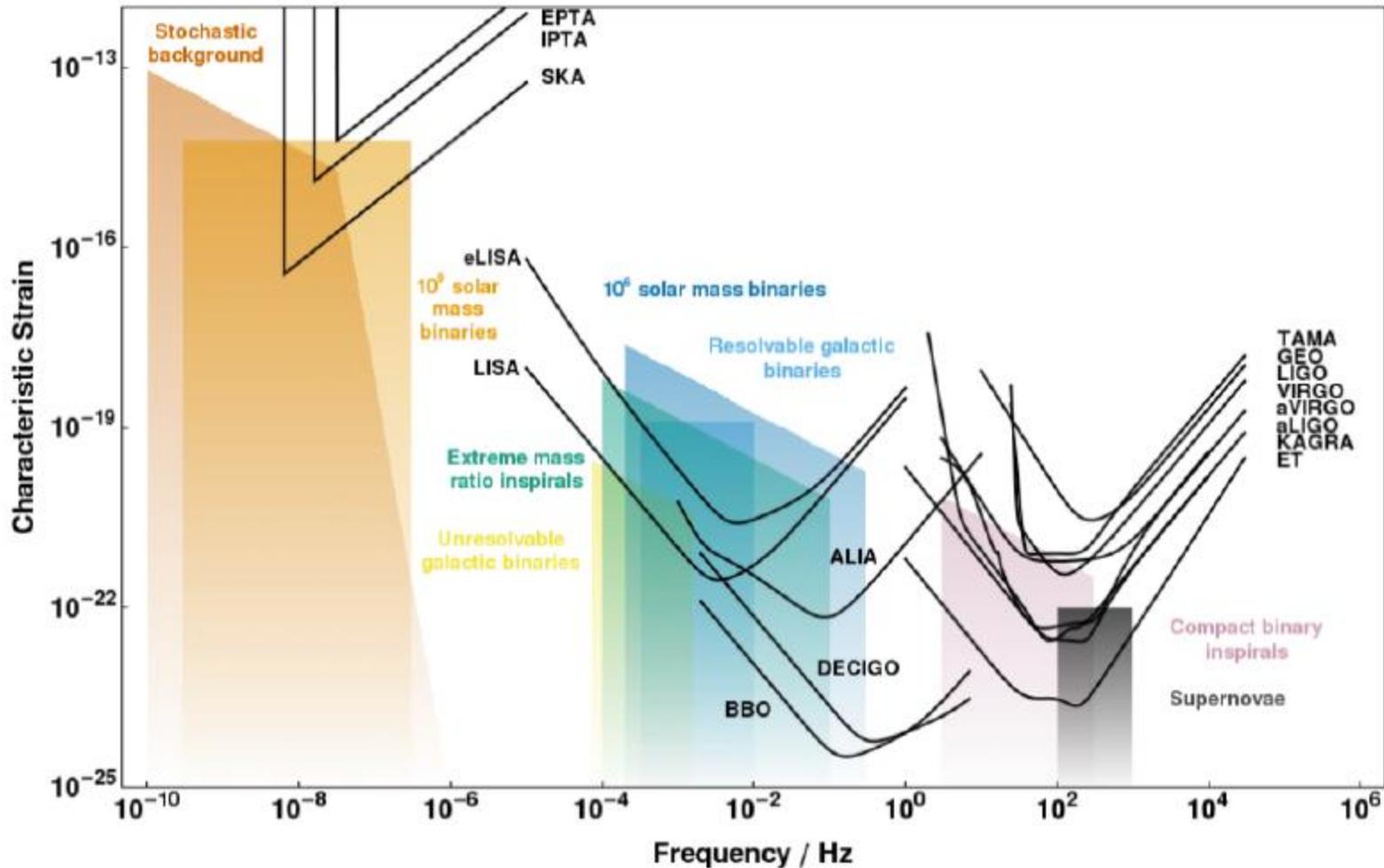
$$h_+(t) = h_0 \left(\frac{1 + \cos^2 \iota}{2} \right) \cos \Phi(t); \quad h_\times(t) = h_0 \cos \iota \sin \Phi(t),$$

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} \epsilon f^2}{d} \quad \text{Abbott et al. 2007}$$

ϵ is oblateness parameter

- For a B-WD of $M=2M_\odot$, $R=2000\text{km}$, $\epsilon \sim 5 \times 10^{-4}$, $P \sim 10$ sec, at 100pc , $h_0 \sim 10^{-23}$: DECIGO/BBO can probe
- For $R=7000\text{km}$, $P \sim 20\text{sec}$, at 10pc , $h_0 \geq 10^{-22}$: LISA can probe
- For an B-WD of $R=700\text{km}$, $P \sim 1$ sec, at 100 pc $h_0 \sim 10^{-22} \rightarrow$ even LIGO can detect in principle

Sensitivity



<https://smirshekari.wordpress.com/2014/04/28/plotting-the-sensitivity-curves-of-gravitational-waves-detectors/>

AR Scorpii to be a seed of B-WD

- White dwarf/cool star binary emitting from X-ray to radio
- Pulsing in brightness on a period 1.97 min
- Maximum luminosity $L \sim 6.3 \times 10^{32}$ erg/s
- Mean Luminosity $L \sim 1.7 \times 10^{32}$ erg/s
- Mass is 0.8-1.29 M_{\odot}
- Spin-down power: $L_{\nu} = 4\pi^2 I \dot{P} / P^3$
- For a typical neutron star $L_{\nu} \sim 10^{28}$ erg/s
- For white dwarfs with radius 2200-7000km
 $L_{\nu} \sim 1.5 \times 10^{32} - 10^{33}$ erg/s
- Mean luminosity excess over stellar contribution
 $\sim 1.3 \times 10^{32}$ erg/s $\rightarrow L_{\nu}$ is sufficient to explain this for a white dwarf but not for a neutron star
- ✓ Suggesting AR Sco primarily a spin-powered white dwarf pulsar: Marsh et al., 2016, Nature, 537, 374

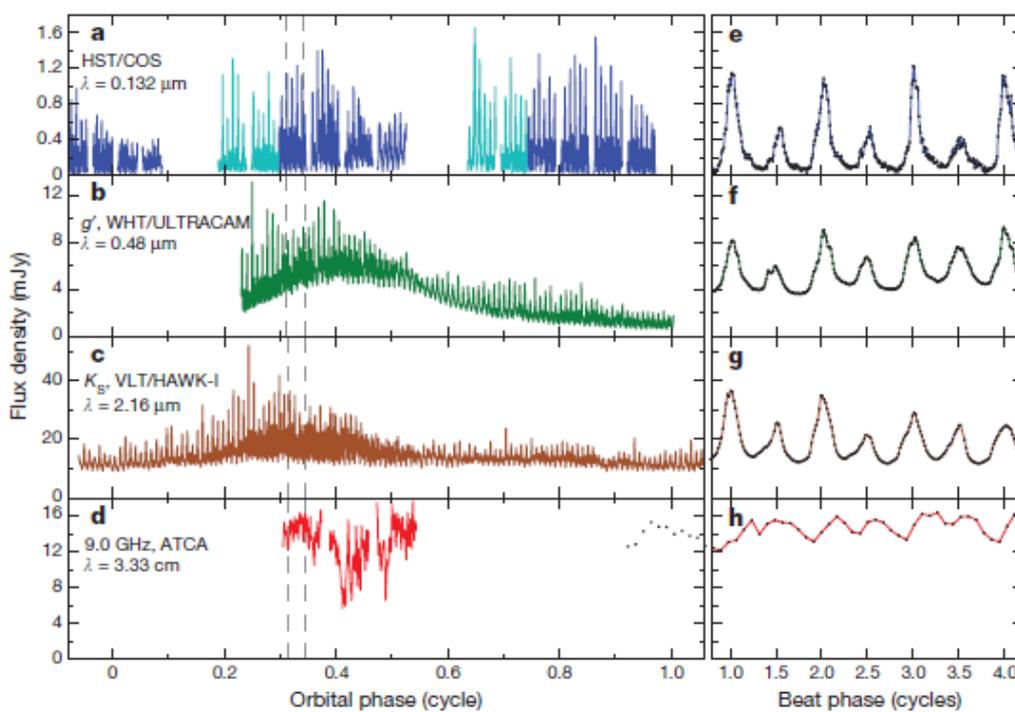


Figure 2 | Ultraviolet, optical, infrared and radio fluxes of AR Sco. a–d, High-speed measurements of the ultraviolet (a), optical (b), infrared (c) and radio fluxes (d) of AR Sco plotted against the orbital phase. e–h, An expanded view of sections of similar orbital phases (marked by dashed grey lines in a–d), is plotted against the beat pulsation phase. Black dots mark individual measurements. None of the four sets of data were taken simultaneously in time. The different colours in a indicate that the data were acquired in different orbital cycles.

First time a white dwarf is found with radio and far-infrared emissions

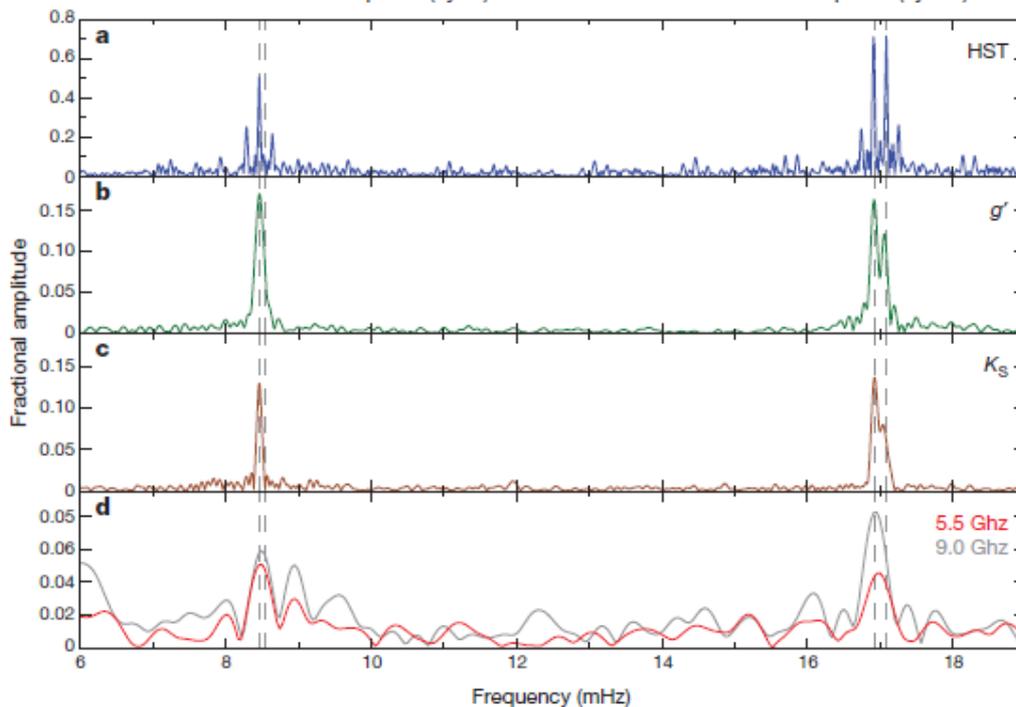


Figure 3 | Fourier amplitudes of the ultraviolet, optical, infrared and radio fluxes of AR Sco versus temporal frequency. a–d, Amplitude spectra corresponding to Fig. 2a–d. All bands show signals with a fundamental period of about 1.97 min (8.46 mHz) and its second harmonic. The signals have two components, clearest in the harmonic, which we identify as the spin frequency ν_S and beat frequency $\nu_B = \nu_S - \nu_O$, where ν_O is the orbital frequency. The pairs of grey dashed lines mark the positions of the beat (left) and spin (right) frequencies and their second harmonics. The beat component is the stronger of the two and defines the dominant 1.97-min pulsation period; the spin period is 1.95 min.

Possible Evolution of AR Sco to B-WD

- Perhaps, initially it was accreting → R decreases, B increases (flux freezing) → suddenly rotates fast due to conservation of angular momentum (releasing stress due to sudden decrease of moment of inertia) → AR Sco → accretion is inhibited → P increases → luminous
- After 10^7 yr, B decays and P increases and radiation stops
- By gravitational wave and angular momentum loss, binary shrinks, accretion starts again → whole cycle repeats
- Repeating cycles will have lower and lower P → ending up as a fast spinning B-WD (SGR/AXP) → Astrosat/LAXPC may detect corresponding electron cyclotron absorption line

Evolution of AR Sco

BM, Rao, Bhatia, MNRAS 2017

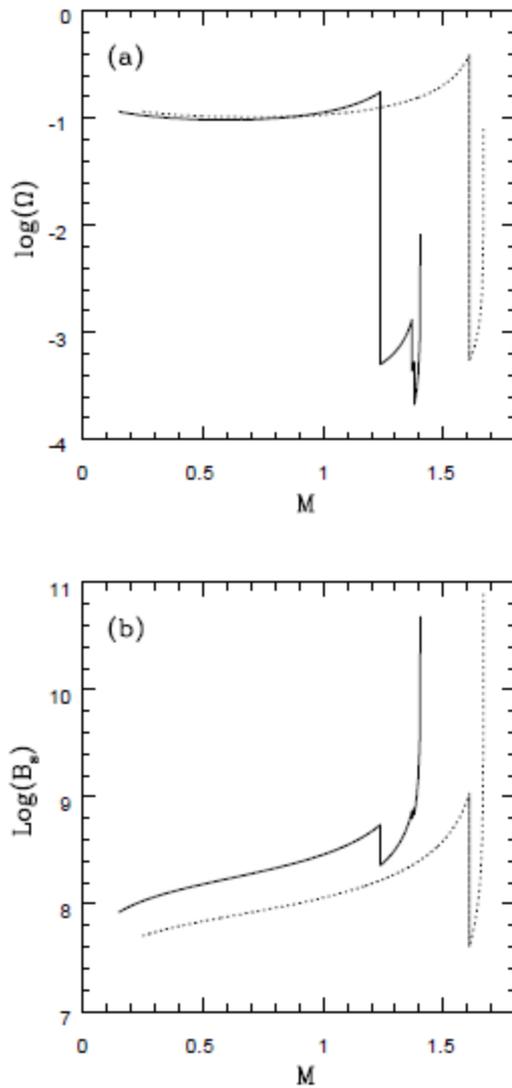


Figure 2. Time evolution of (a) angular velocity in sec^{-1} , (b) magnetic field in G, as functions of mass in units of solar mass. The solid curves correspond to the case with $n = 3$, $m = 2.7$, $\rho = 0.05 \text{ gm cm}^{-3}$, $l = 1.5$ and dotted curves correspond to the case with $n = 3$, $m = 2$, $\rho = 0.1 \text{ gm cm}^{-3}$, $l = 2.5$. Other parameters are $k = 10^{-14} \text{ CGS}$, $\dot{M} = 10^{-8} M_{\odot} \text{ Yr}^{-1}$, $\alpha = 10 \text{ degree}$ and $R = 10^4 \text{ km}$ at $t = 0$.

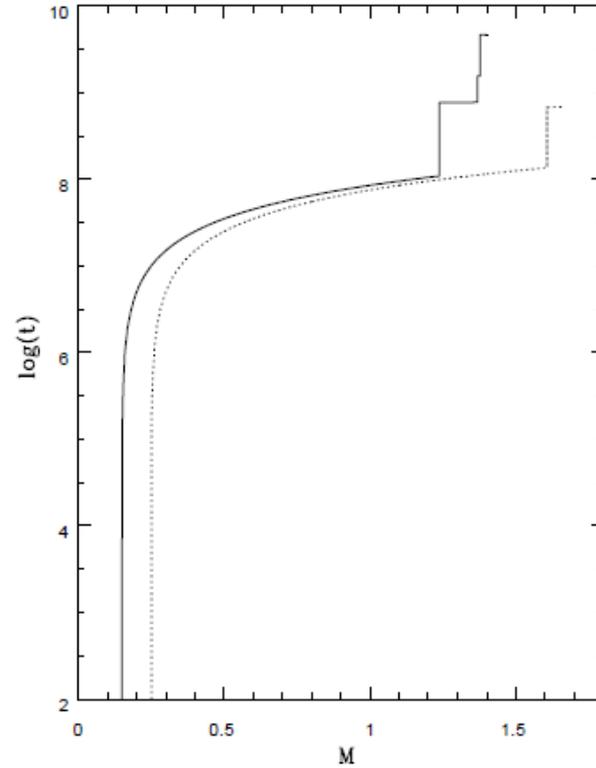


Figure 4. Time taken in Yr to evolve the mass and magnetic fields of white dwarfs shown in Fig. 2.

Accretion phase

$$l\Omega(t)^2 R(t) = \frac{GM(t)}{R(t)^2},$$

$$I(t)\Omega(t) = \text{constant},$$

$$B_s(t)R(t)^2 = \text{constant},$$

$$-\frac{GM}{R^2} = \frac{1}{\rho} \frac{d}{dr} \left(\frac{B^2}{8\pi} \right) \Big|_{r=R} \sim -\frac{B_s^2}{8\pi R\rho},$$

Spin-powered phase

$$\Omega = [\Omega_0^{1-n} - k(1-n)(t-t_0)]^{\frac{1}{1-n}},$$

$$B_s = \sqrt{\frac{5c^3 I k \Omega^{n-m}}{R^6 \sin^2 \alpha}},$$

Summary and Conclusions

- Highly magnetized stable white dwarfs are possible with varieties of application
- New, generic, mass limit of white dwarfs seems to be around $2.6M_{\odot}$
- Once the limiting mass is approached, the white dwarfs explode exhibiting over-luminous, peculiar type Ia supernovae: inferred exploding mass $2.3 - 2.8 M_{\odot}$
- **Suggesting second standard candle: established for certain magnetized white dwarfs**
- They serve as a very good candidate for SGRs/AXPs with less fields
- LAXPC/AstroSat would help in determining their fields and hard X-ray tail
- **In gravitational wave (GW) search they should be considered**
- AR Sco be a proto B-WD: on accretion, seed, apparently dormant field, could be enhanced due to flux-freezing and leading to a B-WD