Simulations of Solar and Stellar dynamos

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1. Introduction
The solar cycle

- Besides the 11 yr cycle
- ~80 yr periodicity (Gleissberg, 1973)
- ~2 yr periodicity (Fletcher et al. 2010)
- Mounder-like minimum events
P(-) → T(-) → P(+) → T(-) → ...

- Where is (are) the solar dynamo (s) located?
- Do sunspots form from magnetic flux tubes or from MHD instabilities near photosphere?
Two branches: A (active) and I (inactive) stars. Is this trend real?

Several stars in the sample exhibit 2 cycle periods: e.g., N/n, E/e, G/g, J/j

Is the Sun a solar-like star?

Is this evidence of multiple dynamos operating in the stellar interiors?

Stars across the HR diagram

\[
\langle |B_v| \rangle \propto R_o^{-1.2}
\]

\[
\langle |B_v| \rangle \propto R_o^{-1.38 \pm 0.14}
\]

Vidotto et al. (2014)
Is there a fundamental dynamo mechanism able to explain the saturation of magnetic field strength?

Do stellar tachoclines play a role in this dynamo process?
2. Modeling stellar dynamos
Mean-field dynamo mechanism
Parker (1955)

- Induction equation

\[ \frac{\partial B}{\partial t} = \nabla \times (U \times B - \eta J) \quad J = \nabla \times B \]

\[ \nabla \times (U \times B) = - (U \cdot \nabla B) + (B \cdot \nabla U) - (B \nabla \cdot U) \]

advection \quad stretching \quad compression

- Induction/advection vs. diffusion

\[ R_m = \frac{u_{\text{rms}}}{(\eta k_f)} \]

<table>
<thead>
<tr>
<th>( T [K] )</th>
<th>( \rho [\text{g cm}^{-3}] )</th>
<th>( P_m )</th>
<th>( u_{\text{rms}} [\text{cm s}^{-1}] )</th>
<th>( L [\text{cm}] )</th>
<th>( R_m )</th>
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<tbody>
<tr>
<td>Solar CZ (upper part)</td>
<td>( 10^4 )</td>
<td>( 10^{-6} )</td>
<td>( 10^{-7} )</td>
<td>( 10^6 )</td>
<td>( 10^8 )</td>
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<tr>
<td>Solar CZ (lower part)</td>
<td>( 10^6 )</td>
<td>( 10^{-1} )</td>
<td>( 10^{-4} )</td>
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<tr>
<td>Protostellar discs</td>
<td>( 10^3 )</td>
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<td>CV discs and similar</td>
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<td>( 10^{-7} )</td>
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<tr>
<td>AGN discs</td>
<td>( 10^7 )</td>
<td>( 10^{-5} )</td>
<td>( 10^4 )</td>
<td>( 10^5 )</td>
<td>( 10^9 )</td>
</tr>
<tr>
<td>Galaxy</td>
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<td>( 10^{-24} )</td>
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<td>( 10^{20} )</td>
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<tr>
<td>Galaxy clusters</td>
<td>( 10^8 )</td>
<td>( 10^{-26} )</td>
<td>( 10^{29} )</td>
<td>( 10^8 )</td>
<td>( 10^{23} )</td>
</tr>
</tbody>
</table>
\[ \frac{\partial \vec{B}}{\partial t} - \frac{1}{r} \left[ \frac{\partial}{\partial r} \left[ r \left( u_r + \gamma_r \right) \vec{B} \right] + \frac{\partial}{\partial \theta} \left[ \left( u_\theta + \gamma_\theta \right) \vec{B} \right] \right] = s \left( \vec{B}_p \cdot \nabla \right) \Omega \]

\[ - \left[ \nabla \eta_T \times (\nabla \times \vec{B}) \right]_\phi + \eta_T \left( \nabla^2 - \frac{1}{r \sin \theta} \right) \vec{B} + \left[ \nabla \times (\alpha^D \vec{B}) \right]_\phi \]

\[ \frac{\partial \vec{A}}{\partial t} - \frac{1}{s} \left[ (\vec{u}_p + \gamma_p) \cdot \nabla \right] (s \vec{A}) = \eta_T \left( \nabla^2 - \frac{1}{r \sin \theta} \right) \vec{A} + (\alpha^D \vec{B})_\phi \]

with \( \eta_T = \eta + \beta \) The total magnetic diffusivity

\[ C_\alpha = \frac{\alpha_0}{\eta_T k_f}, \quad C_\Omega = \frac{\Delta \Omega}{\eta_T k_f^2}, \quad C_U = \frac{U_0}{\eta_T k_f} \]
**Ω-effect (P → T)**

Obtained by helioseismology inversions

(Schou et al. 1998)

Azimuthal flow of differential rotation

The longer the arrow the faster the flow

Meridional magnetic field is transformed into azimuthal magnetic field
\[
\frac{\partial \bar{B}}{\partial t} - \frac{1}{r} \left[ \frac{\partial}{\partial r} \left[ r (u_r + \gamma_r) \bar{B} \right] + \frac{\partial}{\partial \theta} \left[ (u_\theta + \gamma_\theta) \bar{B} \right] \right] = s (\bar{B}_p \cdot \nabla) \Omega
\]

\[-[\nabla \eta_T \times (\nabla \times \bar{B})]_\phi + \eta_T \left( \nabla^2 - \frac{1}{r \sin \theta} \right) \bar{B} + [\nabla \times (\alpha^D \bar{B})]_\phi\]

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\]
\( \alpha \)-effect (P \( \rightarrow \) T): contribution from MHD turbulence

\[ \alpha_{ij} \dot{B} = \alpha B \]

and \( \alpha \) is a pseudo-scalar. It can only exist if the system lacks of reflectional symmetry (e.g., the system is rotating).
\[
\frac{\partial \vec{B}}{\partial t} - \frac{1}{r} \left[ \frac{\partial}{\partial r} \left[ r(u_r + \gamma_r) \vec{B} \right] + \frac{\partial}{\partial \theta} \left[ (u_\theta + \gamma_\theta) \vec{B} \right] \right] = s(\vec{B}_p \cdot \nabla) \Omega \\
\left[ \nabla \eta_T \times (\nabla \times \vec{B}) \right]_\phi + \eta_T \left( \nabla^2 - \frac{1}{r \sin \theta} \right) \vec{B} + \left[ \nabla \times (\alpha^D \vec{B}) \right]_\phi
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\frac{\partial \vec{A}}{\partial t} - \frac{1}{s} \left[ (\vec{u}_p + \gamma_p) \cdot \nabla \right] (s \vec{A}) = \eta_T \left( \nabla^2 - \frac{1}{r \sin \theta} \right) \vec{A} + (\alpha^D \vec{B})_\phi
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\]
$\alpha$-$\Omega$ dynamo with solar differential rotation

(A) $\Omega(r, \theta)$ Isocontours

(B) $\alpha\Omega$ Model, $\alpha \sim \sin^2 \theta \cos \theta$, $C_\alpha = +10$

(C) $\alpha\Omega$ Model, $\alpha \sim \sin^2 \theta \cos \theta$, $C_\alpha = -10$

Charbonneau (2010)
After an educated (not always possible) fine tuning of parameters

- Deep meridional flow
  - Nandy & Choudhuri (2002)
  - Chatterjee et al. (2004)

- $\alpha$ effect in two different locations
  - Dikpati et al. (2004)

- Turbulent pumping
  - Kitchatinov & Olenskoi (2011)

See also: Bonanno et al. (2002), Jouve et al. (2008), Käpylä et al. (2006)
Global simulations, solve the full set of MHD equations in 3D and spherical geometry.
Difficulties

\[ \text{Re} = \frac{u_{\text{rms}} L}{\nu} \sim 10^{12} \quad (10^2) \]

\[ \text{Rm} = \frac{u_{\text{rms}} L}{\eta} \sim 10^9 \quad (10^2) \]

\[ \text{Ra} = \frac{G M (\Delta r)^4}{\nu \kappa R^2} \frac{-1}{c_p} \frac{ds}{dr} \geq 10^{15} \quad (10^6) \]

- Important dynamical scales go from km’s to hundreds of Mm.
- Energy transfer from bottom to top
- Simulations use large values for dissipative terms to keep stable
- Large-scale fields evolve in time scales going from years to decades
- Simulations take long time to achieve HD and MHD steady states

- SGS parametrization is needed
  Guizaru et al. (2010), Guerrero et al. (2016), Auguston et al. (2015)
Dynamo simulations with EULAG-MHD

\[ \nabla \cdot (\rho_s u) = 0, \quad (2) \]

\[ \frac{Du}{Dt} + 2\Omega \times u = -\nabla \left( \frac{p'}{\rho_s} \right) + g \frac{\Theta'}{\Theta_s} + \frac{1}{\mu_0 \rho_s} (B \cdot \nabla)B, \quad (3) \]

\[ \frac{D\Theta'}{Dt} = -u \cdot \nabla \Theta_e - \frac{\Theta'}{\tau}, \quad (4) \]

\[ \frac{DB}{Dt} = (B \cdot \nabla)u - B (\nabla \cdot u), \quad (5) \]

- ILES: implicit large eddie simulations, maximizes Re and Rm (see Strugarek et al. 2015)
- Energy equation solves for \( \Theta' \) (stolen from meteorology)
- Able to resolve a tachocline
- Ghizaru et al (2010) was the first global MHD simulation able to produce magnetic cycles
Are tachoclines relevant?

Models **CZ** (convection zone only)

Models **RC** (radiative/convective zones)
The dynamo period

\[ T_{\text{rot}} = 28 \text{ days, } T_{\text{cyc}} = 1 \text{ yr} \]

\[ \eta_t = \frac{1}{3} \tau_c u_{\text{ms}}^2, \]

\[ \eta_0 \]

Turbulent magnetic diffusivity

- **Tachocline Instabilities**: develop turbulence in the stable layer
  
  - (To be confirmed by other codes)

Guerrero+ (2016a)
Sun continues being an odd star

Strugarek et al. (2017)
Models with convective zone only

Guerrero et al. (2017, in prep)
Models with tachocline
Dynamo saturation

- The results of global simulations resemble the regimes observed
- Local dynamo sources vs deep seated dissipation rate
- Meridional circulation and magnetic buoyancy do not contribute to the dynamo process
Conclusions

- Tachoclines play a relevant role in turbulent global dynamos
- Dynamo periods in RC models increase with the rotation period (the Sun is still a weird star)
- The dynamo saturation relation, $B$ vs $Ro$ is reproduced in RC models
- Shear-current instabilities at the tachocline might determine the dynamo cycle.
Thanks!